Optimal design of plates with cell type hollow core

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Abstract. This research relates to the composite sandwich plywood plates with skin layers of birch plywood and a core of straight and waved plywood cell-type ribs. This specific form of ribs allows to simplify manufacturing processes, to increase the glued area and the ways of load transferring paths and to tailor the stiffness in both (longitudinal and transversal) directions providing increased specific stiffness, strength or load bearing capacity (stiffness, strength or load bearing capacity to mass ratio). The various results depending on chosen variables (according to strength-stiffness criteria) were obtained for one span plate in bending. A various thicknesses of plywood sheets are taken for skins and straight rib parts while for waved part of ribs the 3 layer plywood was taken.

1. Introduction

The plates with cell type hollow core can be used in civil engineering as roof, ceiling or walls as well as furniture elements which are subjected to bending. For the rational use of material it is important to choose the optimal skin layer, structure of core layer and bond material. Several cores [1] have been used to improve the stiffness properties of these plates. Ribbed plates with straight ribs have been investigated for past decade and earlier showing advantages comparing to standard plywood panels or standard sandwich panels. Relatively well known planar corrugated plates [2] and the honeycomb plates [3] have been investigated for past few decades, although mostly for aluminium or other metallic material.
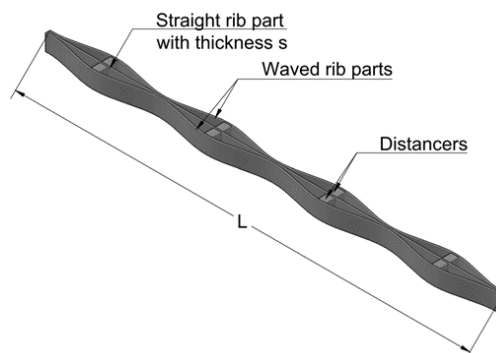

Figure 1. Cell type hollow rib.

Recently investigated cell type core (one hollow cell-type rib see figure 1) [4] [5] [6] [7] show the comparison or methodology of calculations of the structure although the optimization of the thickness and the core structure for such plates should be developed and it is set as a main objective of this paper.

The ribs are placed and glued together with each other and the skins. The plates with cell type hollow core is more resistant to the moisture changes [8] inside the plate due to their geometry (the grain direction of plywood for the core) comparing to standard plywood or cross laminated timber plates.

2. Methodology

In this paper the methodology for plate's element optimization was developed in the expression of metamodeling in a form of global polynomials. Since the serviceability limit state is determinant for such plates - the stiffness (bending stiffness *EI* and shear stiffness *GA*) has the highest influence on allowable load level (functions *f* and *g*). Function *f* will be dependent on bending moment (*M*) while the function g will be dependent on shear force (*Q*) which allow to use the methodology for various loading types. In this case the statically determined structures in 4-point bending have been analysed.

$$u_{inst} = \frac{f}{EI} + \frac{g}{GA} \le u_{lim} \qquad (1)$$

where u_{inst} - plate deflection, u_{lim} - limited deflection according to serviceability limit state (1/200 of plate's span).

To optimize the structures of plate, the required bending stiffness should be provided at minimum consumption of materials. The consumption of materials can be taken into account by using Area Mass Density (AMD) to use and compare the optimum results for various geometry, boundary conditions and loadings. The objective function and the approximation variables should be defined. If it is necessary to decrease the number of variables then the influence of variables should be analysed and the parameters with less influence should be eliminated for optimization by fixing these values. The experimental design depending on these information and additionally adapted ANSYS APDL program code with Probabilistic Design (PD) functions has been created for calculation of plates with cell type hollow core at the design points. The achieved response values are exported back to design software for approximation. The multi-criterion optimization can be done by given parameter boundaries and defined restrictions for response functions.

2.1. Optimization algorithm

The optimization algorithm is shown in figure 2. It consists of two parts – Experimental design, approximation and FEM analysis. The approximation and optimization are realized by the EdaOpt software [9] while the FEM analysis has been done with ANSYS APDL and methodology algorithm for calculation of plates with cell type hollow core.

1) *Definition of the problem* - The limitations have been defined for each of variable depending on optimization task – geometrical parameters of skins and core and the physical material properties, load cases and levels, etc.

2) *Design of experiments* - The design of experiments should be done depending on their limitations and discretion. By creation of experimental design the limitations that is due to material options (e.g. plywood manufacturer nominal thicknesses), technological possibilities as well as dimensions of a plate (e.g. the length of a wave will be dependent on number of waves along the span and the width of a rib will be dependent of width of a plate. If the number of variable is small (<5) then full factor experimental design could be used, although by increasing the number of variables, the experimental points increase exponentially. In that case one of advanced experimental designs or other optimization methodology (e.g. Genetic Algorithm) should be used.

3- 5) *Experimental design plan, calculations and the preparation of output file* - ANSYS APDL input file (written in .txt format) that is based on methodology of calculation for plates with cell type hollow core. The code was read into the program and after the solution the probabilistic design solver was called to use simulations for given experimental plan to obtain the points for response approximation in .pdrs format that can be edited like a .txt file.

6– 8) *Response function and the validation of the results* - The most widely used response surface method with 2nd power polynomial for approximation was chosen. Global mean square error criterion

was used to approximate the function with regression coefficients and determined by the minimization of the sum of squared differences of the design points and approximate values.

9) *Multi-criterion optimization and the end of algorithm* - Multi-criterion optimization should be done by using approximated response functions (minimizing or maximizing related functions) for the given parameters and their boundaries.

After the validation of optimization the optimal parameters have been saved and the designed plate can be used in engineering calculations and the solution of manufacturing technology.
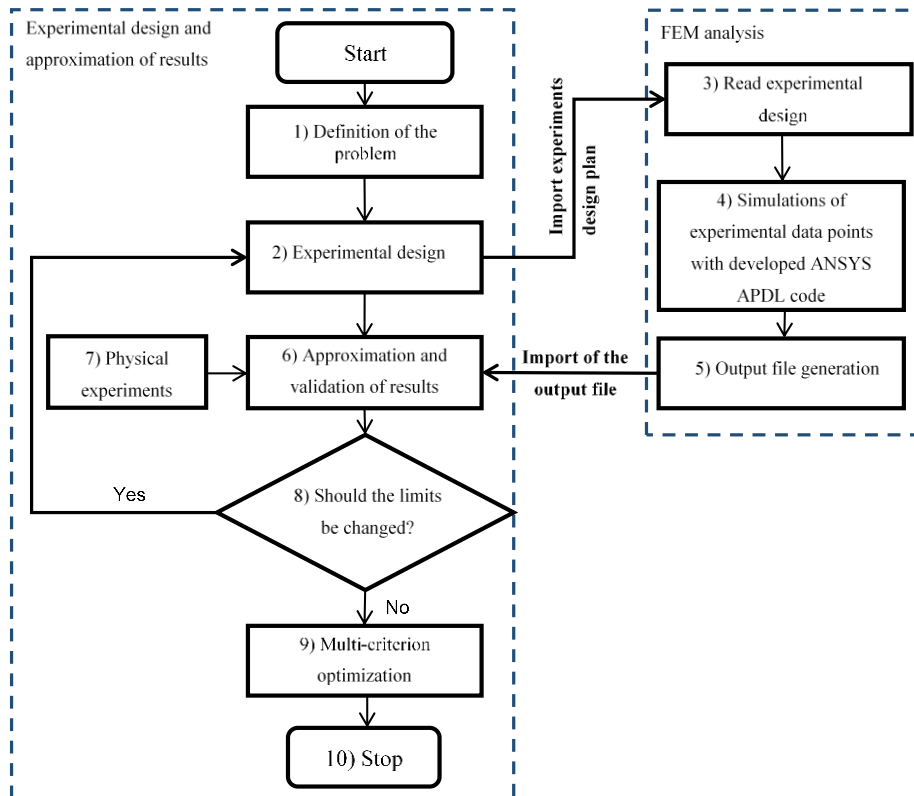

Figure 2. Optimization algorithm with experimental design.

2.2. *Experimental investigations*

The experimental investigations to validate the methodology of calculations is done (step 7). Several design points obtained from FEM (ANSYS V.15 [10]) simulations have been compared (figure 3) and experimentally validated to the results achieved by guiding to LVS EN789:2005 [11].
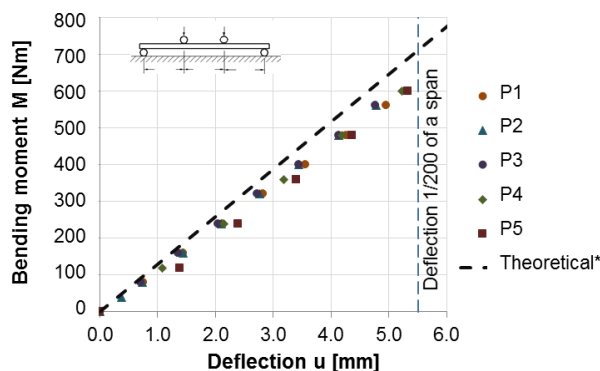

Figure 3. Bending moment/deflection for plates (Specimens P1 - P5) with span 1.1 m (guiding to LVS EN 789:2005 *[11]*) and the straight rib elements with thickness of 6.5 mm and waved rib part 4.0 mm. *- Theoretical achieved from ANSYS simulations reaching serviceability limit state.

The statistical analysis showed the mean value of stiffness is 51050 Nm² with standard deviation 1655 Nm²; and coefficient of variation 3.39%; confidence interval of mean value for probability of 95% is ±1450 Nm². These results are the approval that for stiffness determination the distribution is acceptable and as well as satisfying coincides to numerical investigations has been achieved.

2.3. The influence of optimization parameters
The choice of optimization methodology is depending on complicity of the problem (the objective function and characteristics of it), number and type of variables and properties. To improve the approximation it is recommended to evaluate the parameters influence on the response function and reduce the number of variables to eliminate the unnecessary calculations while running the experimental design [12]. The serviceability limit stage is the determinant in most of the cases for such plates [7]. The influence of load bearing capacity is determined depending on thickness-to-span ratio in initial stage of calculations. If it has not been defined other way then the variables have been assumed like this: Length of a plate - 1.2 m, thickness of skins - 4.0 mm and direction of outer plies parallel to the span direction, width of rib - 60 mm and the straight rib part - 6.5 mm and waved rib part - 4.0 mm.

In this research it has been assumed that bending load bearing capacity for statically determinant systems is dependent on bending moment (M_{max}) while the shear component of the deflection does not exceed 12% and has been ignored for optimization. And the will be optimized.

The response function gets more complex when the number of variables increases more than five and the discretization of variables decreases. In these cases the additional discretization in recommended if there is no possibility to decrease the number of variables or use of Genetic Algorithm should be considered.
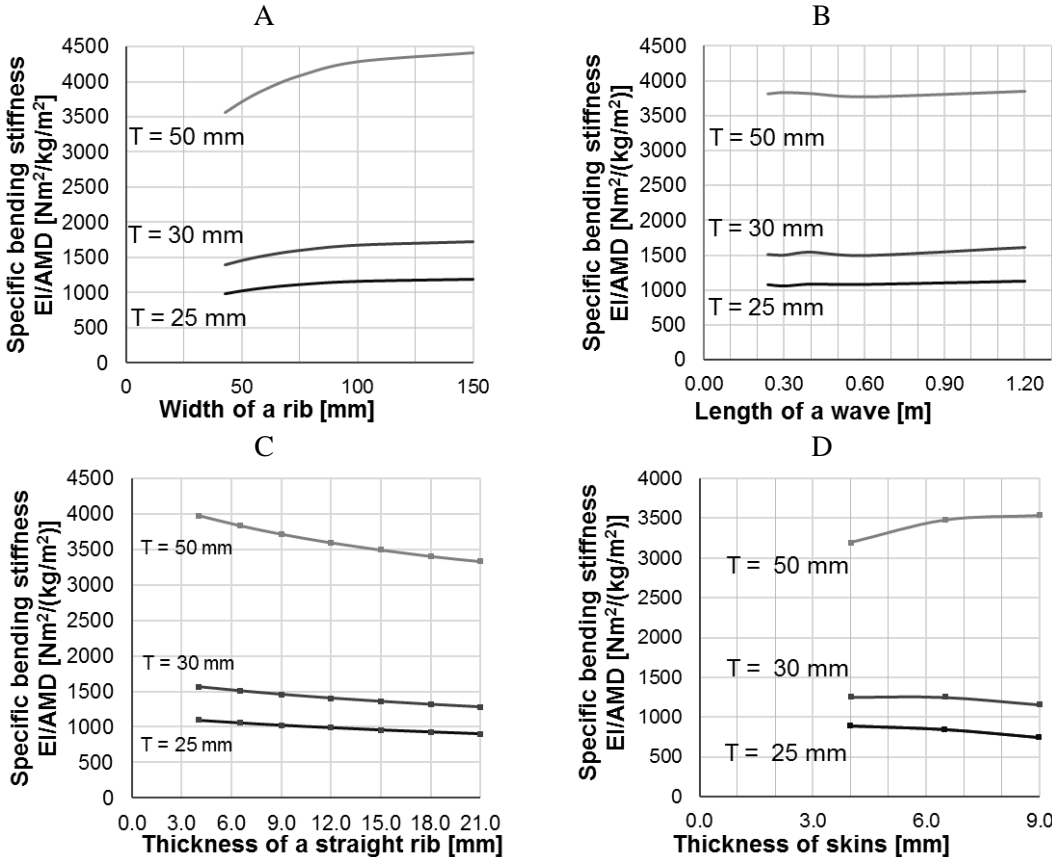

Figure 4. Specific bending stiffness. For plate with span 1.1 m and the rib parts with thickness of 4.0 mm. A – depending on the width of a rib; B - depending on the length of a wave; C- depending on the thickness of straight rib part; D – depending on the thickness of the skins.

The influence of variables has been determined. The specific stiffness and the load bearing capacity is highly dependent on the thickness of a plate. Although the other parameters can vary when the thickness of the plate is limited. The influence of the other parameters shown in figure.

The achieved results shown in figure 5. The shear component of deflection was determined as a difference from total deflection and the deflection from pure bending. By changing this ratio the influence of shear deflection changes. Depending on this ratio three groups have been chosen – for ratio up to 1/40 the influence of deflection is mainly from the bending moment, for ratio 1/40 – 1/10 the influence from the shear deflection increases from 5 to 25 % and for the ratio over the 1/10 the influence of shear deflection is higher than 25% and the both (bending moment and the shear force) should be taken into account.
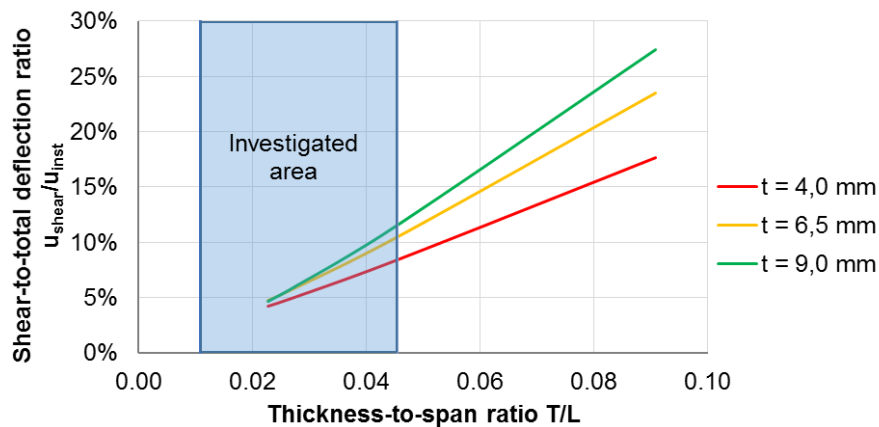

Figure 5. Shear-to-total deflection depending on Thickness-to-Span ratio.

Additional to the thickness of a plate, the thickness of skins and straight rib part and various spans have been chosen. In some cases the thickness of a plate is limited and in that way the requirement for optimization of other plates elements should be done. By minimization of Area Mass Density (*AMD*) at given loading and required stiffness the specific load bearing capacity (*LBC$_{sp}$*) can be written as function of span of the plate *L*, thickness of a plate *T*, thickness of skins *t* and thickness of straight rib part *s*:

$$\max(\text{LBC}_{sp}) = \{L; T; t; s\} \tag{2}$$

Optimization has been done by approximation of stiffness function and calculation the Area Mass Density for relating parameters that provides such stiffness. Since the number of variable has been reduced in this case the full factor experimental design can be applied or space filling experimental design by minding the discretization of several parameters. In this case the regular experimental design has been chosen and for thickness of plywood the number of layers has been determinant for choosing design points. Two different design plans have been chosen (table 1) due to the various boundaries for consisting element or rational use of provides structure for the various thicknesses. The plans have been joined at the top boundary for small thicknesses and the lower boundary for larger thicknesses to compare the results achieved from both approximations.

Table 1. Design space.

Variables	Nomenclature	25 – 50 mm		50 – 150 mm	
		Lower bound	Upper bound	Lower bound	Upper bound
Thickness of skins [m]	t	0.0040	0.0090	0.0065	0.0180
Thickness of straight rib part [m]	s	0.0040	0.0210	0.0090	0.0210
Thickness of a plate [m]	T	0.025	0.050	0.050	0.150
Length of a plate [m]	L	1.2	2.4	2.4	6.0

To determine the stiffness and mass values for various geometrical parameters given in table 1 for fixed width of the ribs and waved rib parts, the polynomial can be rewritten as follows:

$$EI_i = b_{i,0} + b_{i,1} \cdot L + b_{i,2} \cdot T + b_{i,3} \cdot t + b_{i,4} \cdot s + b_{i,11} \cdot L^2 + b_{i,12} \cdot L \cdot T$$
$$+ b_{i,13} \cdot L \cdot t + b_{i,14} \cdot L \cdot s + b_{i,22} \cdot T^2 + b_{i,23} \cdot T \cdot t + b_{i,24} \cdot T \cdot s \qquad (3)$$
$$+ b_{i,33} \cdot t^2 + b_{i,34} \cdot t \cdot s + b_{i,44} \cdot s^2$$

If i=1 then polynomial for thickness 25-50mm; i=2 for thicknesses 50-150mm.

In all cases the ultimate limit state in plate consisting elements has been checked and determined that the serviceability limit state is reached before the ultimate limit state (the maximum load factor reaching the serviceability limit state does not exceed 53% in any of cases). Therefore the optimization has been done for area mass density minimization influenced only by the required bending stiffness.

2.4. Approximation

The approximation was made for bending stiffness depending on geometrical parameters of core and compared for various spans for given requirements. The coefficients of approximation equation shown in Table 2.

Table 2. Coefficients for equation.

		25mm<= T <= 50mm	50mm<= T <=150 mm
Const.	[Nm^2]	2.94×10^4	4.94×10^5
$b_{i,1}$	[Nm]	4.37×10^0	5.11×10^4
$b_{i,2}$	[Nm]	-2.77×10^6	-1.85×10^7
$b_{i,3}$	[Nm]	1.18×10^6	-8.67×10^6
$b_{i,4}$	[Nm]	-3.17×10^5	-1.38×10^7
$b_{i,11}$	[N]	-2.07×10^2	-9.64×10^3
$b_{i,12}$	[N]	4.82×10^4	5.23×10^5
$b_{i,13}$	[N]	1.36×10^5	1.10×10^6
$b_{i,14}$	[N]	6.99×10^3	1.24×10^6
$b_{i,22}$	[N]	4.98×10^7	1.12×10^8
$b_{i,23}$	[N]	1.63×10^8	4.94×10^8
$b_{i,24}$	[N]	2.76×10^7	3.12×10^8
$b_{i,33}$	[N]	-3.33×10^8	-3.94×10^8
$b_{i,34}$	[N]	-6.25×10^7	-6.96×10^8
$b_{i,44}$	[N]	5.45×10^4	8.94×10^6

2.5. Experimental validation and adequacy check

It has been achieved that 95% of approximate values are in a range of 5% from the design points for the thicknesses 25 - 50 mm and 90% of values are in range of 5% for the thicknesses from 50 to 150 mm which approves that the obtained polynomials are adequate for the use of calculation and optimization of load bearing capacity of plates. The thickness-to span ratio is in range of 1/92 to 1/22 in which the shear influence on load bearing capacity does not exceed 15 % (figure 5). The adequacy check for each experimental design shows that coefficient of determination D (or R –squared), which is used for measuring multifactor correlation is 0.99 for both cases that approves the results are reliable and can be used for optimization of ribbed plates.

3. Results

By choice of experimental design it is recommended to consider the amplitude of response function values. In general the minimization of larger absolute error is by larger response values can lead to relatively larger errors by smaller responses in that way the approximation is not adequate in these

regions. In these cases the local weighted mean square should be applied or the reduction of variable boundaries should be done.

3.1. Comparison of plates with various increase of thickness

The obtained approximation was used for optimal design of plates with cell type hollow core to achieve the maximal specific stiffness in longitudinal direction for various thicknesses of a plate. As the thickness of a plate has the most influence on load bearing capacity (and stiffness), the optimization procedure chooses to increase this value until the required stiffness is obtained. Approximate values shown in table 3. As a result the deflection of the beam is depending on bending moment and the stiffness from equation (1). The load bearing capacity can be calculated by finding moment M when maximum allowable deflection u_{fin} is reached. And finally the specific load bearing capacity can be found by dividing maximal allowable bending moment (M) with related AMD. The approximation has been made for plate with span of a plate L=2.0 m, length of a wave l = 300 mm, width of a rib b = 60 mm.

Table 3. Specific load bearing capacity comparison of plates for various thicknesses.

		thickness increased for 10%			thickness increased for 10%		
T [mm]	M_{max}/AMD [Nm²/kg]	T* [mm]	M_{max}/AMD [Nm²/kg]	Difference[1]	T_{pl}** [mm]	M_{max}/AMD [Nm²/kg]	Difference[1]
25.0	46.2	27.5	55.6	21%	30.0	65.54	42%
30.0	65.5	33.0	78.0	19%	36.0	91.01	39%
35.0	86.6	38.5	102.3	18%	42.0	118.91	37%
40.0	109.3	44.0	128.7	18%	48.0	149.22	36%

[1] - Difference is shown comparing specific load bearing capacity to the initial load bearing capacity
* - the plates with cell type hollow core with
** - the plates with cell type hollow core with increased thickness of 20%;

Analogue procedure could be done for transversal direction of a plate if there are additional requirements for stiffness in transversal direction or for different supporting type.

3.2. The sequence of Optimization

The research provides two possibilities of optimization: the optimization of plate's parameters if the load bearing capacity is defined for a construction element or if the stiffness is defined for the design of a plate if it is defined as a material.

If the problem is defined as a design of material in general (e.g. for manufacturing) then the specific stiffness should be maximized by minimizing the mass of the plates. For example, if the required stiffness is 51000 Nm², then the optimal thickness of a plate is 48.50 mm with the thickness of skins and straight rib part of 4,0 mm which gives the maximal specific stiffness 3990 [(Nm²)/(kg/m²)]

If the problem is defined as a required load bearing capacity (this case he maximal bending moment M_{max}) then the optimization should be done for the maximal specific bending load bearing capacity. For the given bending moment value the optimization for the structural elements of the plate should be done for limited thickness of a plate, which has been fixed to the value of 27 mm. The required load bearing capacity has been set to 180 Nm. The optimization shows: when the maximum limited thickness of plate has been reached, the thickness of the skins is the next parameter to be optimized. After they both have reached the maximum value for given design space, the other values (thickness of the straight rib part or number of ribs) will be optimized. This remains the same for all thicknesses of the plate although the local deformation effects should be taken into account when calculating such plates to approve additional advantage comparing these plates to the plates with only straight ribs or plates with only curved ribs. As a result then the optimized thickness of a plate is 27 mm, thickness of skins 6.5 mm and the thickness of straight rib part 4.0 mm.

4. Conclusions

Two approaches have been developed for the optimization of cell type hollow core. The optimization can be done for structural element which provides the required load bearing capacity with the minimal consumption of materials. However, when the plate with cell type hollow core is defined as a material then the specific stiffness can be optimized by finding the required structure of plate's elements.

The influence of geometrical parameters of consisting element of plates with cell type hollow core on the specific stiffness was investigated. The analysis shows that the most influence has the thickness of the plate, but in cases where the thickness of plate is limited the thickness of the skins and thickness of the straight rib part can be increased to increase the stiffness of a plate.

As the response function is highly dependent of thickness of a plate it is recommended to divide the design space into discrete regions where the difference between the highest and the lowest response value is smaller to minimize the square error differences for the design points near to upper and lower boundaries.

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