

## Non-linear buckling analysis of steel frames with local and global imperfections

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The problem related to nonlinear analysis of steel frames is analyzed by taking into account local and global imperfections. A new method for estimation of optimal (in the worst case scenario causes smallest buckling load) pattern for geometrical and material imperfections is developed. The method is based on the optimization of structural topology. The topology is optimized in a way where maximal compliance (minimal stiffness) and strain energy is reached. According to the gradient of compliance function the pattern of geometrical and material imperfections is iteratively obtained. Global geometrical imperfections are applied by using buckling modes (Eigenmodes) with a pre calculated weight coefficient according to the compliance function for each buckling mode. The proposed methods showed to be robust and practically applicable for designing complex steel structures.

**KEYWORDS:** Steel frames, buckling, imperfection's pattern, structural optimization.

### Introduction

Steel structural members are not geometrically perfectly shaped due to manufacturing and building tolerances. Three types of initial imperfections should be taken into account in advanced (second-order inelastic) analysis: (i) the member out-of-straightness (bow imperfection), (ii) the frame out-of-plumb (sway imperfection) and (iii) local material imperfections (Shayan et. al. 2014, Papp 2016) such as deviation in Young's modulus or residual stress. The pattern of initial imperfections is often chosen to be the worst case scenario to maximize their destabilizing effects and compliance under the applied loads in a global frame analysis.

The modelling of geometric and material imperfections is more complicated for a frame (or any spatial 3D structure) than for a single column because not only the magnitude but also the pattern (shape and the direction) of the imperfection affects the overall response of the frame.

Papadopoulos et. al. (2013) proposed a method for accounting portal frame's stochastic imperfections that are described with non-homogeneous Gaussian random fields by using brute Monte Carlo simulations. A similar method is used to take into account the effect of initial geometrical imperfections on the buckling load of steel tubes under axial load and lateral pressure (Vryzidis et.al. 2013). This method is computationally too intensive and requires many parameters to describe the Gaussian field of each type of imperfection, therefore it may be too complex for a practical engineer.

Special attention is paid to the buckling analysis of cold formed steel structures. As shown in the work done by Garifullin and Nackenhorst (2015), C- shaped cold formed steel structures experience very complicated buckling and post-buckling behavior due to initial geometrical imperfections. These imperfections arise from their manufacturing process, shipping, storage and construction process. The geometric imperfections of cold formed steel members can be represented by the member eigenmode shapes. Along with the classical measure (the amplitude of imperfections) an energy measure defined by the square root of the elastic strain energy hypothetically required to distort the originally perfect structural element into the considered imperfect shape – can be used (Sadovský et. al. 2012). A non-linear buckling analysis of 2-D cross-aisle storage rack frames may be done using shell finite elements providing an idea about optimal structure (Thombare et.al. 2016). As shown in work done by Zeinoddini and Schafer (2012), the imperfection measurements for cold-formed steel members can be simulated using (i) superposition of eigenmodes, (ii) multi-dimensional spectral representation or (iii) a combination of the modal approach and spectral representation.

Welding process can cause a significant residual stress and imperfections in the geometry. As shown in the publication done by Sadowski et.al. (2015), the spiral welded carbon steel tubes may have a very unique pattern of surface imperfections. Prediction of flexural buckling of stainless steel I-section and box-section columns may be problematic with current design standards (Yang et.al. 2016a, Yang et.al. 2016b).

Some works are done to define the equivalent geometric imperfection for analysis of steel structures that are sensitive to flexural and/or torsional buckling due to compression or bending moment (Agüero et.al. 2015a, Agüero et.al. 2015b). It is shown that beam element analysis can be used to account for second order effects in locally and/or distortionally buckled frames (Zhang et.al. 2016b). This method is validated with experimental investigations (Zhang et.al. 2016a). Meanwhile, additional bending moments may arise due to geometrical global and local imperfections (Baláz and Koleková 2012). There is still on-going research on the smaller structural scale (meso scale), where computational

homogenization methods are coupled with analysis of geometrical imperfections (Goncalves et. al. 2016). The effect of mode interaction between local buckling and global buckling about the strong axis in thin-walled I-section struts can be taken into account with a simple analytical model (Liu and Wadee 2016).

Although extensive research has been conducted on advanced analysis for steel structural systems (Shayan et. al. 2014), a robust mathematical method of estimation of initial geometric and material imperfection's pattern in advanced non-linear analysis has not been developed. For this propose a novel method of estimation of material and geometrical imperfection patterns that can cause the maximal compliance of the structure is proposed. The maximal compliance indicates that the structure has minimal stiffness and it can be maximally easy to deform with a specific load case. This will also provide the worst buckling scenario and minimal buckling load. The material imperfections may be a local deviation of Young's modulus and local cross section imperfection. In this paper proposed a method that will take into account the local imperfections by performing structural topology optimization in the sense of maximizing structural compliance. Meanwhile, there is also proposed a method for accounting for the global geometric in-plane imperfections that is based on the Euler buckling analysis and a special method that combines the worst buckling modes to get the final imperfection's pattern. In this study, 2D steel frames are analyzed. However, this method can be easily extended to a 3D case. This method can be easily implemented into a commercial structural engineering software.

### Estimation of local imperfection's pattern

Typically, structural topology optimization is used to optimize structures for maximal stiffness (minimal compliance) (Tsavdaridis et.al. 2015, Sliseris and Rocens 2013), but in this case done contrary by maximizing the structural compliance. This will indicate locations in the structure that should be made weaker by applying imperfections to achieve worst scenario and minimal stiffness (maximal compliance) of the structure. In this case, we use structural topology optimization that seeks for the optimal deviation of Young's modulus in different locations of the structure to reach minimal stiffness (maximal compliance). Optimization is based on 2D linear Euler-Bernoulli beam finite element analysis.

Young's modulus of  $e$ -th element  $E_e$  is assumed to be a function of the imperfections density  $\rho(e)$  given by the linear interpolation scheme as:

$$E_e = E_0(1 - \rho(e)), \quad (1)$$

where  $E_0$  is Young's modulus for material without local imperfections (210 GPa is used in this work).

The structural topology optimization problem is formulated to maximize the compliance  $C$  and strain energy

$$\begin{cases} \text{find } \rho(x) \\ \text{max } C = U^T(x)K(x)U(x) \\ \text{s. t. } \begin{cases} K(x)U(x) = F \\ \sum_{e=1}^N \frac{\rho_e v_e}{V_0} - 1 \leq 0 \\ 0 < \rho_{min} \leq \rho_e \leq \rho_{max} \end{cases} \end{cases}, \quad (2)$$

where  $U(x)$  is the displacement vector,  $K(x)$  global stiffness of structure,  $F$  global force vector,  $N$  number of finite elements,  $v_e$  volume of  $e$ -th finite element,  $V_0$  reference volume of structure that cannot be exceeded,  $\rho_{min}$  minimal value for imperfection's density,  $\rho_{max}$  maximal value of imperfection's density.

The values of imperfection density can be indirectly estimated from design codes (e.g. Eurocode 3) or standards.

The Gaussian filter  $G$  is used in the optimization to smooth the final values of imperfection density function by calculating convolution

$$\rho(x) = \int_0^t \rho^*(x)G(t-x)dx, \quad (3)$$

where  $\rho^*(x)$  is non-smoothed vector of imperfection's density.

The Gaussian smoothing function  $G(t-x)$  is given in a standard form:

$$G(t-x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-x)^2}{2\sigma^2}}, \quad (4)$$

where  $\sigma$  indicates the width of the filter (in this work the width was equal to length of 3 finite elements).

### Estimation of global geometrical imperfection's pattern

In this paper is presented a method that is based on the work Shayan et.al. (2014). This method is extended in a more mathematical way. First of all, the linear static analysis is performed to calculate the axial forces in each member. In the next step, the linear-buckling eigenvalue problem is solved

$$K_e \phi_i = \lambda_i K_g \phi_i, \quad \lambda_i = P_{cri}, \quad i = 1, \dots, n, \quad (5)$$

where  $K_e$  global stiffness matrix,  $K_g$  geometrical stiffness matrix,  $\lambda_i$  eigenvalues corresponding to the critical loads  $P_{cri}$ ,  $\phi_i$  is buckling modes and  $n$  corresponds to number of degrees of freedom.

Buckling modes are scaled using a defined maximal global geometrical imperfection parameter  $\delta^{max}$

$$\phi_i^* = \phi_i \frac{\delta^{max}}{\max(\phi_i)}. \quad (6)$$

$\delta^{max}$  can be estimated by design standards (e.g. Eurocode 3) or experimental measurements (Sadowski et.al. 2015).

The next step is to calculate structural compliance for each scaled buckling mode  $\phi_i^*$ . The structural geometry is changed to the scaled buckling mode and static analysis is performed

$$K(\phi_i^*)U(\phi_i^*) = F. \quad (7)$$

$$C_i = U(\phi_i^*)^T K(\phi_i^*) U(\phi_i^*), \quad i = 1, \dots, n. \quad (8)$$

The final geometrical imperfection's pattern  $\Phi$  is obtained by calculation of weight coefficients for each buckling mode corresponding to the structural compliance:

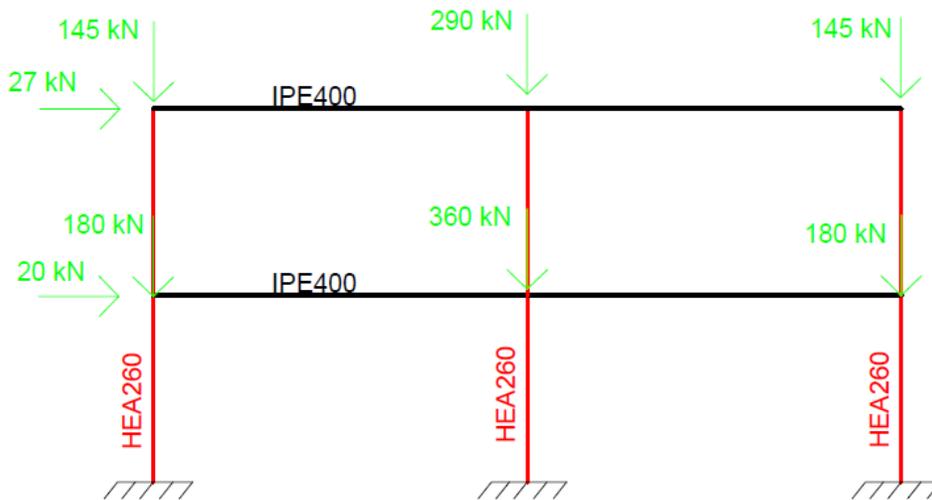
$$\Phi = \sum_{i=1}^M \phi_i^* \frac{C_i}{\sum_{k=1}^n C_k}, \quad (9)$$

where  $M$  corresponds to the number of buckling modes that should be taken into account for the final imperfection's pattern calculation.

The proposed methods are implemented into 64-bit Matlab R2015a programming environment.

## Results

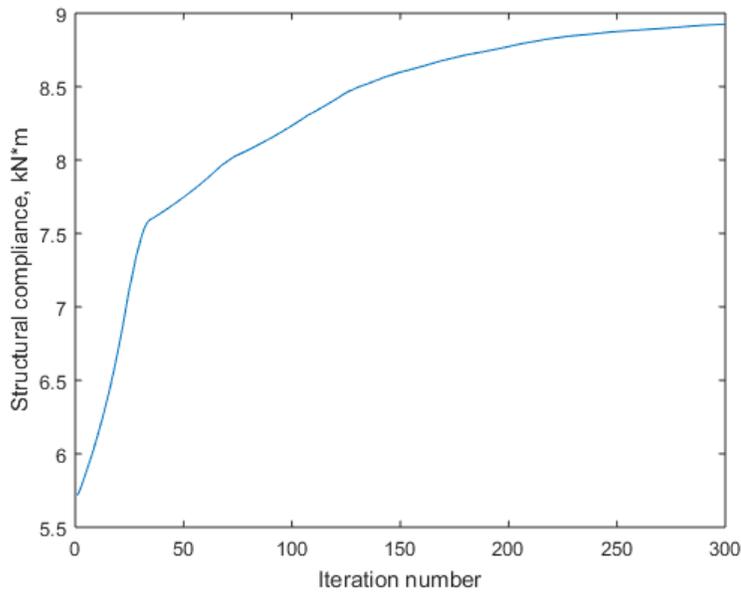
A typical 2-story frame with 2 spans (see Fig. 1) is analysed to show the performance of the proposed methods. The columns are made of HEA 260 structural steel I-sections with length of 5 m. Beams are made of IPE 400 structural steel I-sections with span of 10 m. The modulus of elasticity (without imperfections) is 210 GPa. The boundary conditions and applied loads are shown in the Fig.1. The entire structure is meshed with beam finite elements with size 0.5 m. The beam finite element is based on classical Euler-Bernoulli beam theory.



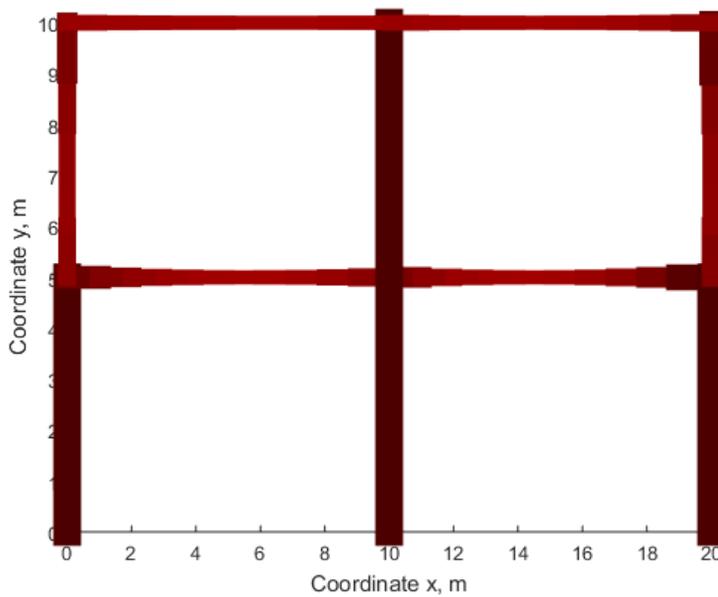
**Fig. 1.** Scheme of the 2D typical frame

First of all, we estimated the optimal topology of local imperfections (e.g. deviations of Young's modulus or local cross section imperfections). The proposed method was successfully used and showed a good convergence (see Fig.2.). The convergence was reached in 250 iterations. In the Fig.3. the topology of imperfections is shown. The thickness of member indicates the sensitivity of overall structural compliance to local imperfection in the member. As we can see, the middle columns have very large influence on the global stiffness and compliance of the frame. The horizontal members may not have an in-plane buckling (out of plane, torsional and flexural buckling are not considered in present work).

This work indicates that for a simple 2D frame structure it may be possible to find the pattern of local imperfections by "hand" calculations considering members with the highest axial force. However, for a spatial 3D frame structure this method may significantly improve the methodology to identify the worst scenario (pattern of local imperfections). In future work this method will be extended to 3D cases and a large spatial steel structure will be analysed.



**Fig. 2.** Structural compliance.

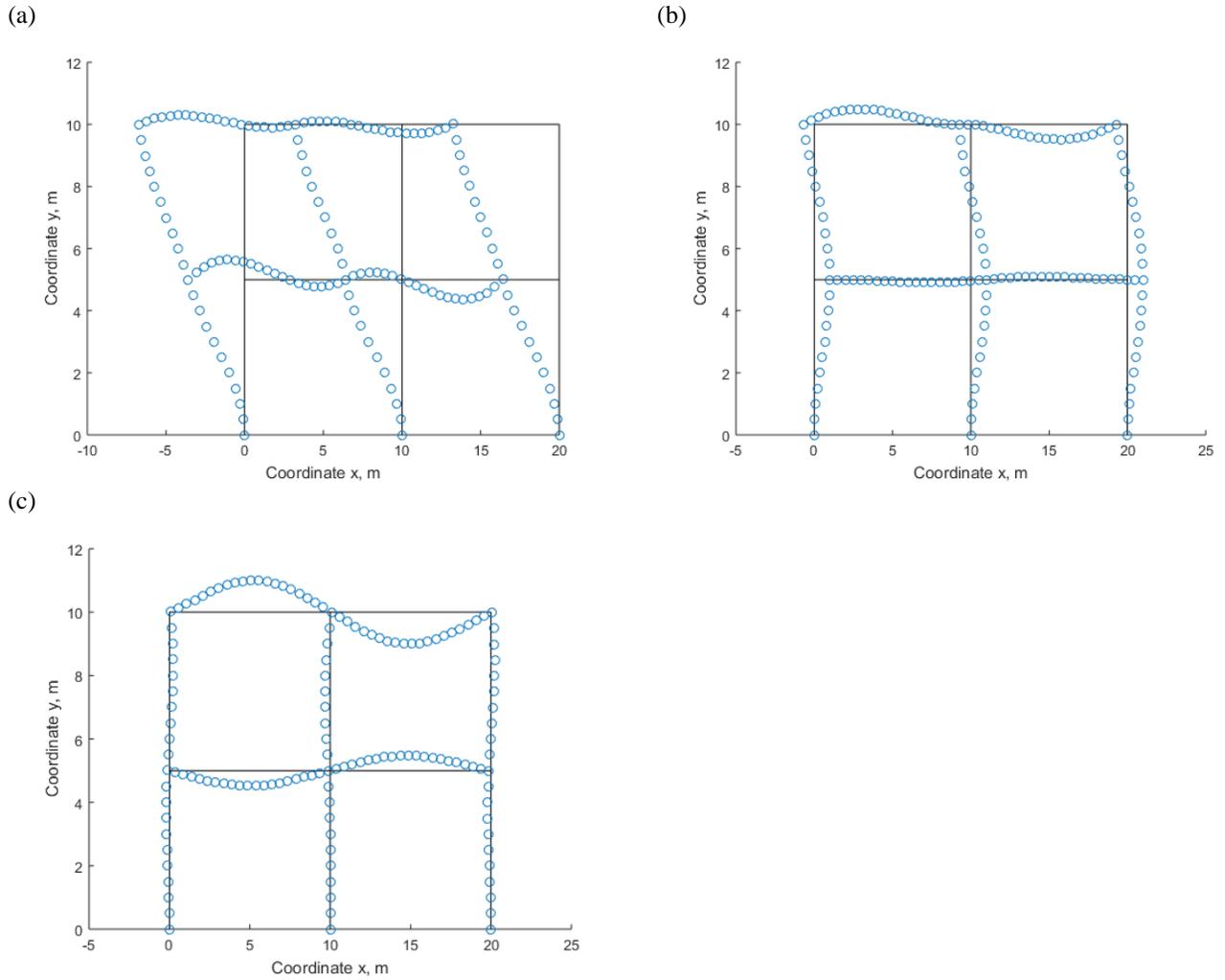


**Fig. 3.** Imperfection’s topology for material Young’s modulus (or local cross section imperfections) corresponding to maximal compliance of the frame. Thickness of the line corresponds to value  $1 - \rho(e)$ .

The pattern of global geometrical imperfections that can cause a worst case scenario is estimated for the same frame that is shown in Fig.1. The first three buckling modes are shown in Fig.4. The proposed method is used to estimate the influence of each buckling mode on structural compliance. The Fig. 5. indicates that actually the first buckling mode is not the one that can cause maximal compliance. The second and fifth buckling mode show the maximal compliance (minimal stiffness) of the frame.

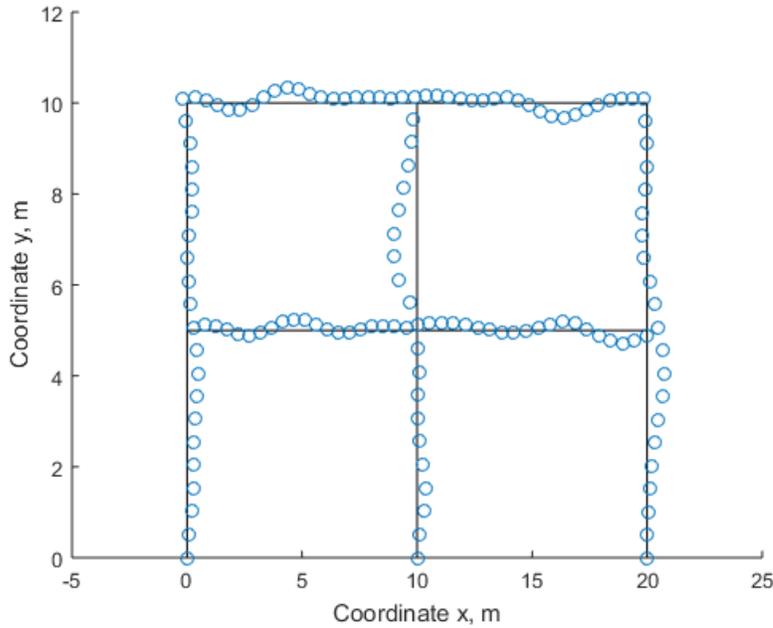
The proposed method was used to estimate the worst pattern of global geometrical imperfections by considering (i) first 10 buckling modes (see Fig. 6), (ii) first 50 buckling modes ( see Fig. 7) and (iii) first 100 buckling modes). The simulations showed that higher buckling modes did not have a realistic shape (that can be observed in experimental investigations) and therefore were not taken into account.

The research is still in progress and in succeeding works the proposed method will be extended to estimate the weight coefficient for each buckling mode in a more robust mathematical method using a compliance-gradient based topology optimization approach.

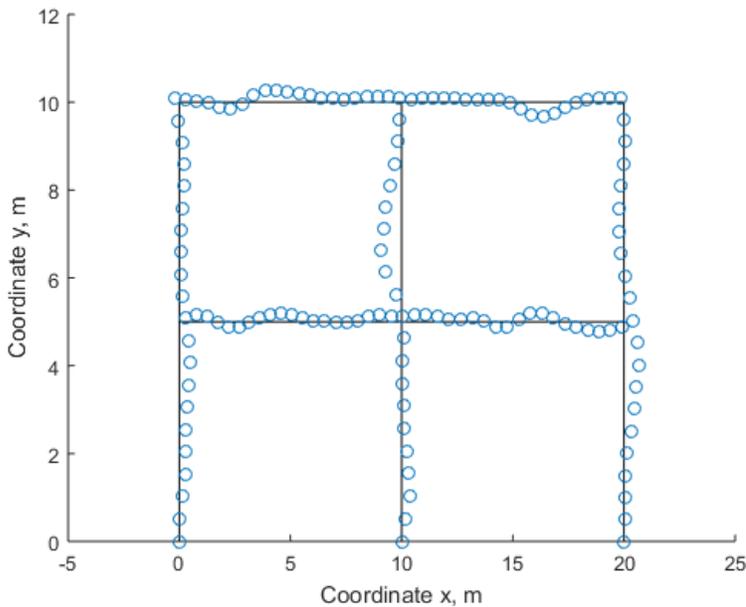


**Fig. 4.** Typical buckling modes of the frame. (a) First buckling mode, (b) second buckling mode, (c) third buckling mode.





**Fig. 7.** Final shape with geometrical imperfection's pattern when first 50 buckling modes are considered.



**Fig. 8.** Final shape with geometrical imperfection's pattern when first 100 buckling modes are considered.

## Conclusions

A novel method for estimation of patterns of local and global geometrical imperfections that can cause worst case scenarios is proposed. The local imperfection's pattern is estimated by using the structural topology optimization approach and seeking for the pattern that can cause a minimal stiffness (maximal compliance) of the overall structure. The pattern of global geometrical imperfections that can cause a minimal stiffness of frame is estimated by a special superposition of buckling modes. For each buckling mode a weight coefficient is calculated that depends on the structural compliance for this buckling mode. Both methods can be effectively used in advanced non-linear buckling analysis of frames.

The current work is not finished and the methods will be updated. In the future, the presented methods should be

extended and coupled with non-linear finite element simulations. The global imperfections should be calculated iteratively by assigning weight coefficients for each buckling mode based on the topology optimization approach.

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