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Wavelet transform based damage detection in a plate structure

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Abstract

A 2-dimensional wavelet transform algorithm employing isotropic Pet Hat wavelet is used to locate the area of damage in a numerically simulated aluminium plate model. To study the advantages and limitations of proposed method, signals of vibrational mode shapes of the plate are reduced by integer numbers to study the performance of algorithm at various sensor densities. Results suggest that the lowest scale of selected wavelet function yields the best damage identification results with a damage location likelihood of more than 98 %.

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Keywords: wavelet transform; plate; standardized damage index; mode shape.

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1. Introduction

Demands for structural inspection have grown in recent decades. The need of fast, cheap and effective methods without employment of the significant manpower is a today's prerogative in structural health monitoring (SHM). While a wide range of vibration-based damage identification mechanisms exist, it is also preferable to employ such damage identification methods that do not require a baseline data of vibration response for a healthy structure, such as natural frequencies, mode shapes and damping. Unfortunately, only few of damage identification methods meet this criterion. One of such methods is Wavelet Transform (WT).

WT is a digital signal processing technique that simultaneously provides good resolution in time and frequency for a given signal. This WT capability of managing analysis of continuous as well as transient signals [1] has gained a wide popularity among many engineering communities. Today WT is used in signal discontinuity detection, image

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compression and de-noising, also in medicine and finance. Variations of the WT were extensively employed in fault detection not only in rotating machinery [2], but also in structural engineering, namely, cracks in beams (1D WT) [3-5], plates (2D WT) [6, 7] delamination, debonding in composite structures [8-13].

In this paper, the area of damage in an aluminium plate was identified with 2D Wavelet Transform technique. The most promising wavelet function turned out to be Pet Hat. Numerically simulated vibrational mode shape signals were corrupted with various levels of noise and reduced by several integer values to simulate the performance of damage detection algorithm in real life situations with different sensor densities.

2. Materials and methods

2.1. Specimens

A square aluminium plate with dimensions of (1000 x 1000) mm and thickness of 5 mm (refer to figure 1) subjected to clamped-clamped boundary conditions at all four edges is considered for this study. The plate is modelled with a commercial finite element program ANSYS. The following values for elastic isotropic material properties are taken: E = 69 GPa, v = 0.31 and ρ = 2708 kg/m^3. The plate is modelled using 8-node shear-deformable shell finite elements. The damage with coordinates 270 mm < x < 350 mm, 220 mm < y < 280 mm is modelled by reducing the flexural stiffness of the selected elements by decreasing the thickness of corresponding elements. The plate is divided into the 100 x 100 elements and the total of 12 mode shapes with the corresponding frequencies are extracted from all 101 x 101 nodes of the plate.

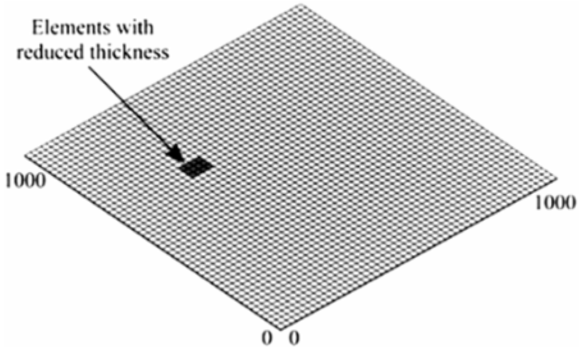

Fig. 1. Numerical simulation of square aluminium plate with the zone of damage.

2.2. Damage identification algorithm

Mode shapes themselves do not reveal the location of damage, therefore special techniques are required. One of such techniques is Wavelet Transform (WT). Wavelets are special functions $\psi(x)$ with small oscillations such, that their mean is zero. WT is a mathematical method to transform the original signal into a different domain where additional data analysis becomes possible. WT can be employed to analyse signals not only in time domain but in space domain as well. 2D Continuous WT (2D CWT) for a two-dimensional signal $f(x,y)$ is given by [6]

$$W_{s,a,b} = \frac{1}{\sqrt{s \cdot s}} \iint f(x,y) \cdot \psi^{*}\left(\frac{x-a}{s}, \frac{y-b}{s}\right) dxdy = \iint f(x,y) \cdot \psi^{*}{}_{s,a,b}(x,y) dxdy \qquad (1)$$

where asterisk denotes complex conjugation and $\psi_{s,a,b}(x,y)$ is a set of wavelet family functions, derived from a mother wavelet function $\psi(x,y)$ by translating (parameters a and b) and dilating (parameter s - scale) the $\psi(x,y)$. Wavelet scale is a real and positive number. If $0 < s < 1$, the function is expanded, if $s > 1$, it is compressed.

Equation (1) is used to calculate 2D CWT coefficients, that are extremely sensitive to any discontinuities and singularities, present in the signal $f(x,y)$, therefore location of damage due to a sudden loss of stiffness can be detected in the mode shapes that yield large amplitude wavelet coefficients.

Damage index for each of mode shapes is depicted as follows

$$DI^{n}_{i,j\,2DCWT} = W^{n}_{i,j} = \iint_{S} w^{n}_{i,j} \cdot \psi^{*}{}_{s,a,b}(x,y)dxdy \tag{2}$$

where S is an area of the plate, w^{n} is a transverse displacement of the structure, n is a mode number, i and j are numbers of grid points in x and y directions, respectively. However, mode shapes, measured in real life experimental conditions, are inevitably corrupted by measurement noise, which can lead to false peaks in damage index profiles, thus misleading data interpreter. In order to overcome this problem, it is proposed to summarize the results for all modes. The summarized damage index is then defined as the average summation of damage indices for all modes N , normalized with respect to the largest value of each mode

$$DI_{i,j} = \frac{1}{N} \cdot \sum_{n=1}^{N} \frac{DI^{n}_{i,j}}{DI^{n}_{i,j\,,\max}} \tag{3}$$

According to [14, 15], the damage indices, determined for each element, are then standardized

$$SDI_{i,j} = \frac{DI_{i,j} - \mu_{DI}}{\sigma_{DI}} \tag{4}$$

where μ_{DI} and σ_{DI} are mean value and standard deviation of damage indices in equation (3), respectively. To quantify the reliability of wavelets to identify damage location, a new parameter, called damage estimate reliability (DER) is introduced and calculated as follows:

- The whole area of the plate is split into 2 parts: part (*a*) – zone of damage:

  270 mm $< x <$ 350 mm, 220 mm $< y <$ 280 mm

  and part (*b*) – the rest of the plate.
- In each of these parts standardized damage indices were summed up and divided by the number of data points in this part giving average amplitude of SDI in the zone of damage and the whole plate.

The following designations were adopted – number of points in x and y direction (x,y) – (I,J) for part (*a*) and (N,M) for part (*b*), respectively. DER in equal to average SDI in the zone of damage divided by average SDI in the whole plate and expressed in percentage as

$$DER = 100\% \cdot \frac{(J \cdot I)^{-1} \sum_{j=1}^{J} \sum_{i=1}^{I} SDI_{i,j}}{(M \cdot N)^{-1} \sum_{j=1}^{M} \sum_{i=1}^{N} SDI_{i,j}} \tag{5}$$

It is often not possible to equip the structure with a dense grid of sensors. Therefore an additional study was conducted where numerical mode shape data was divided by integer numbers $p = 1:8$, leading to mode shape matrices of size $101\text{x}101$, $51\text{x}51$, $34\text{x}34$, $26\text{x}26$, $21\text{x}21$, $17\text{x}17$, $15\text{x}15$ and $13\text{x}13$.

3. Results and discussion

Damage indices were summed over all modes. The sum of all simulated mode shapes for sensor densities, corresponding to values $p = 1,3,5,8$ are shown in figure 2.

The wavelet function was chosen to be a Pet Hat wavelet, which is an isotropic wavelet, meaning transform values do not depend on angles of rotation. In order to find the wavelet scale that yields the best performance of damage identification methodology, analysis of DER values versus scale is performed (refer to figure 3 (a)).

Overall, 32 scales are considered. Best results were attained for 1st scale with a DER value of 98.16 %.

The expression for a Pet Hat wavelet in frequency domain is given in Matlab (MATLAB R2015a) programs` Wavelet Toolbox in the section Continuous Wavelet transform 2-D:

$$\psi\left(\omega_x, \omega_y\right) = \begin{cases} \cos^2\left(\dfrac{\pi \ln\left(\left|\omega_x + i\omega_y\right|\right)}{2\ln(2)}\right) \rightarrow 0.5 < \left|\omega_x + i\omega_y\right| < 2 \\ 0 \end{cases} \tag{6}$$

where ω_x and ω_y are angular frequencies in in x and y direction, respectively. As mentioned in section 2.2, wavelets can successfully be used in spatial analysis. Therefore, as frequency is a reciprocal of time, the wavenumber is a reciprocal of spatial coordinate. Thus, in our case, ω_x and ω_y can be replaced with wavenumbers.

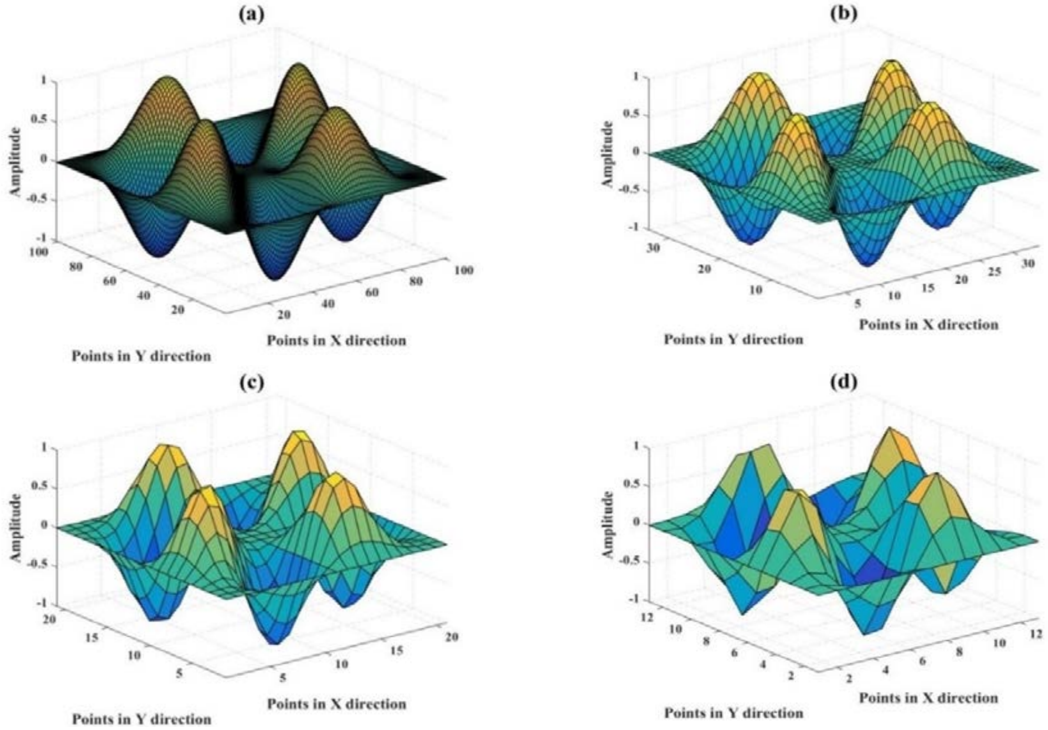

Fig 2. Simulated mode shapes. (a) p = 1, (b) p = 3.

As scale increases, DER has a tendency to decrease. The analysis of the DER versus sensor grid density is shown in figure 3 (b). The highest DER value corresponds to the highest sensor density, whereas as sensor grid density decreases, there is no clear trend for the DER.

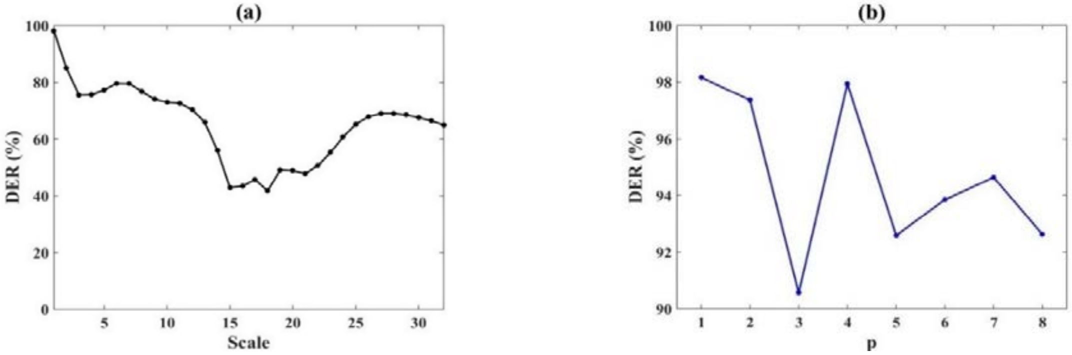

Fig. 3. (a) DER versus scale at $p = 1$. (b) DER versus p at 1st scale.

Thus, only 1st scale is considered for further study. SDI values are computed for all sensor densities. The corresponding SDI contour plots are shown for values $p = 1,3,5,8$ in figure 4.

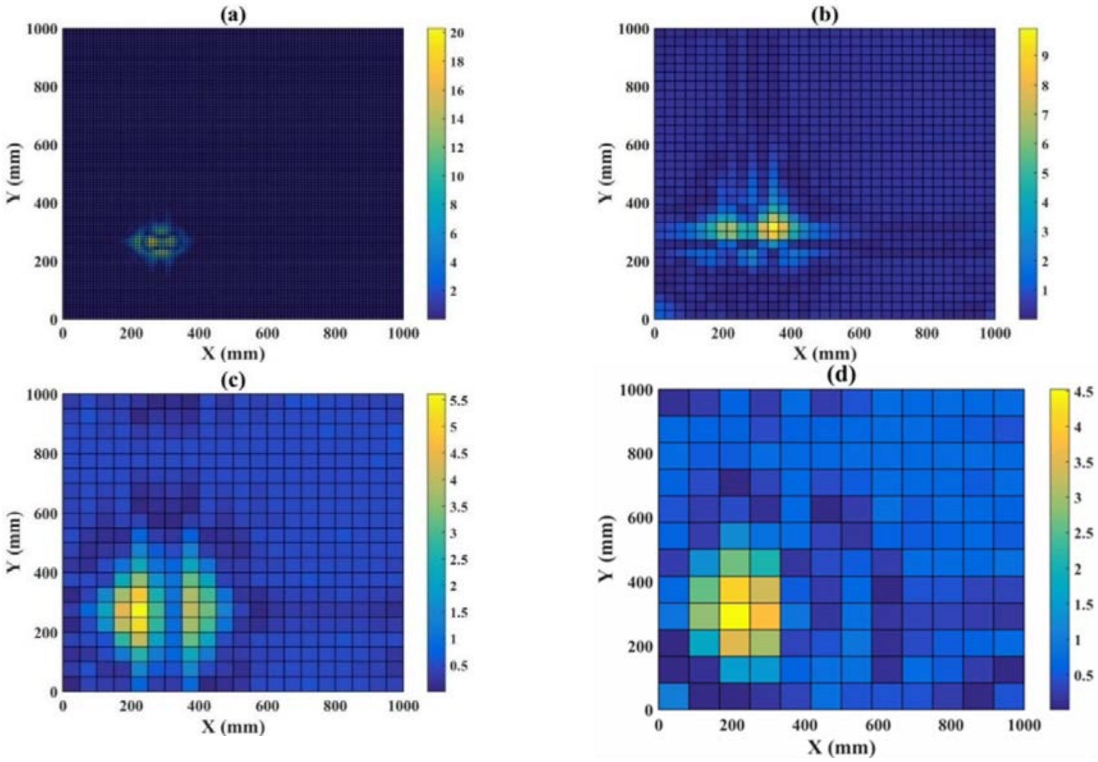

Fig. 4. SDI contour plot at 1st scale. (*a*) $p = 1$, (*b*) $p = 3$, (*c*) $p = 5$, (*d*) $p = 8$.

4. Conclusions

The damage identification based on 2D wavelet transform method is successfully demonstrated on a square aluminium plate containing a mill-cut damage. Damage identification algorithm is tested in numerical simulations in case of damage index sum over all modes for different sensor densities. The likelihood of damage indication for each of these cases is expressed in percentage through damage estimate reliability parameter.

Best damage identification results are achieved using isotropic Pet Hat wavelet, specifically at scale 1. Increase in scale is accompanied by a large damage estimate reliability drop. The proposed damage identification methodology is assessed on varying sensor density, which is simulated by dividing the input mode shape data signal by factors of 1 to 8. While at 1st scale the damage estimate reliability for the densest sensor grid is 98.16 %, and for the coarsest sensor grid – 92.63 %, intermediate sensor grids do not show any clear trend of damage estimate reliability.

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