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Vibration-based approach for structural damage detection

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The idea of the proposed method is based on the relationship between mode shape curvature and the flexural stiffness of a structure. Damage induced reduction in the flexural stiffness of a structure subsequently causes an increase in the magnitude of the mode shape curvature.

Mode Shape Curvature damage index defined by Pandey et al. in 1991

$$MSC_{i} = \sum_{n} \left| \frac{\partial^{2} w_{i,n}^{d}}{\partial x^{2}} - \frac{\partial^{2} w_{i,n}^{u}}{\partial x^{2}} \right|$$

Mode Shape Curvature Square damage index proposed by Ho and Ewins in 2000

$$MSCS_{i} = \sum_{n} \left| \left(\frac{\partial^{2} w_{i,n}^{d}}{\partial x^{2}} \right)^{2} - \left(\frac{\partial^{2} w_{i,n}^{u}}{\partial x^{2}} \right)^{2} \right|$$

However, the major drawback of those methods is a need for the data of the undamaged structure which sometimes can be difficult or even impossible to obtain.



The drawback is solved by either using the finite element model to simulate the dynamic response of the undamaged structure or by employing smoothing techniques to generate a smoothed surface of mode shape curvature obtained from the damaged structure.

$$MSCS_{i} = \sum_{n} \left| \left(\frac{\partial^{2} w_{i,n}^{d}}{\partial x^{2}} \right)^{2} - \left(\frac{\partial^{2} w_{i,n}^{u}}{\partial x^{2}} \right)^{2} \right| \longrightarrow DI_{i} = \sum_{n} \left| \left(\frac{\partial^{2} w_{i,n}^{d}}{\partial x^{2}} \right)^{2} - \left(k_{i,n} \right)^{2} \right|$$

where $k_{i,n}$ represents smoothed surface of mode shape curvature

- Gapped Smothing Technique proposed by Ratcliffe and Bagaria in 1998
- Smoothing Element Analysis proposed by Tesler et al. in 2005
- Locally weighted scatter plot smooth is employed in this study



Damage index

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Damage index generalized to two-dimensional space for the *n*th mode

$$DI_{i,j}^{n} = \left| \left(\frac{\partial^{2} w_{n}}{\partial x^{2}} \right)_{(i,j)}^{2} - \left(k_{x}^{n} \right)_{(i,j)}^{2} \right| + \left| \left(\frac{\partial^{2} w_{n}}{\partial y^{2}} \right)_{(i,j)}^{2} - \left(k_{y}^{n} \right)_{(i,j)}^{2} \right|$$

Central difference approximation

$$\left(\frac{\partial^2 w^n}{\partial x^2}\right)_{(i,j)} = \frac{\left(w_{i+1,j}^n - 2w_{i,j}^n + w_{i-1,j}^n\right)}{h^2} \qquad \left(\frac{\partial^2 w^n}{\partial y^2}\right)_{(i,j)} = \frac{\left(w_{i,j+1}^n - 2w_{i,j}^n + w_{i,j-1}^n\right)}{h^2}$$

The damage index is defined as summation of damage indices for all modes normalized with the respect to the largest value of each mode

$$DI_{i,j} = \sum_{n=1}^{N} \frac{DI_{i,j}^{n}}{DI_{\max}^{n}}$$



Aluminium plate:

Cut depth: 2 mm

Damage parameters:

Dimensions: 1000x1000x5 mm

Location: 270≤*x*≤350 640≤*y*≤700

Vibration experiment setup

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POLYTEC PSV-400-B scanning laser vibrometer:

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PSV-I-400 LR optical scanning head

➢ vibrometer sensor OFV-505

► OFV-5000 controller

► PSV-E-400 junction box

> amplifier Bruel&Kjaer type 2732

computer system with data acquisition board and PSV Software

51 x 51 measurement points





Elements with reduced thickness

1000

Finite Element analysis

1000

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- Five different levels of damage severity
- Four different noise levels

Cut	Noise	Noise	Noise	Noise		
depth,	level	level	level	level		
[mm]	$\delta = 1\%$	$\delta = 0.5\%$	$\delta = 0.1\%$	$\delta = 0.05\%$		
0.5	Case 1.1	Case 1.2	Case 1.3	Case 1.4		
1.0	Case 2.1	Case 2.2	Case 2.3	Case 2.4		
1.5	Case 3.1	Case 3.2	Case 3.3	Case 3.4		
2.0	Case 4.1	Case 4.2	Case 4.3	Case 4.4		
2.5	Case 5.1	Case 5.2	Case 5.3	Case 5.4		

Commercial FE software ANSYS 14.0

0 0

- 52 x 52 eight-node shear deformable shell elements
- Degrees of freedom:

UX, UY, UZ, ROTX, ROTY, ROTZ

- Clamped boundary conditions
- First 15 mode shapes are extracted from 51 x 51 nodes

Uniformly distributed random variables are added to the numerical mode shapes to generate the noise-contaminated mode shapes

$$w^n = \widetilde{w}^n (1 + \delta(2r - 1))$$

where \widetilde{w}^n is noise free transverse displacement of the structure, *r* is the uniformly distributed random values in the interval (0,1), σ is the noise level.



Illustration of the method

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The 12th mode shape of the test plate (a)



Mode shape curvature in *x* direction (b)



17 x 17 measured datapoints



The local regression smoothing process follows these steps for each data point:

1. Compute the *regression weights* for each data point in the span. The weights are given by the tricube function:

$$w_i = \left(1 - \left|\frac{x - x_i}{d(x)}\right|^3\right)^3$$

where x is the predictor value associated with the response value to be smoothed, x_i are the nearest neighbors of x as defined by the span, and d(x) is the distance along the abscissa from x to the most distant predictor value within the span. The weights have these characteristics:

- > The data point to be smoothed has the largest weight and the most influence on the fit.
- > Data points outside the span have zero weight and no influence on the fit.
- 2. A weighted linear least-squares regression is performed.
- 3. The smoothed value is given by the weighted regression at the predictor value of interest.

Matlab R2014a



weighted linear least-squares regression uses a second degree polynomial.





Illustration of the method

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Sum of damage indices for all modes

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5 out of 12 modes

6 out of 12 modes



To evaluate the proposed damage detection index the damage indices determined for each node are standardized and the concept of the statistical hypothesis testing (Bayissa et al., 2008) is used to classify damaged and undamaged elements.

The standardized damage index Z_{ij} at grid point (i,j):

$$Z_{ij} = \frac{DI_{ij} - \mu_{DI}}{\sigma_{DI}}$$

where $\mu_{\rm MSCSM}$ and $\sigma_{\rm MSCSM}$ are the mean value and the standard deviation of the damage indices, respectively.

The decision for the localization of damage is established based on the level of significance used in the hypothesis test which can be determined from a pre-assigned classification criterion.

The typical damage threshold values for the standardized damage index widely used in literature include 1.28, 2, and 3 for 90%, 95%, and 99% confidence levels for the presence of damage.









False-positive damage indication ratio

False-positive indication of damage means that standardized damage indices outside the predetermined damage location have passed the pre-assigned damage classification criterion and indicate the presence of damage although no damage is introduced there.

Ratio represents the relationship between a number of nodes with a false-positive damage indication and the total number of nodes for the plate:

$$R = \frac{\sum_{i,j=2}^{I-1} n(Z_{i,j} \ge 3)}{\sum_{i=1}^{I-1} 1} \times 100\% = \frac{\sum_{i,j=2}^{I-1} n(Z_{i,j} \ge 3)}{(I-2) \times (I-2)} \times 100\%, n(Z_{i,j} \ge 3) = \begin{cases} 1, \text{if } Z_{i,j} \ge 3\\ 0, \text{if } Z_{i,j} < 3 \end{cases}$$

Calculated false-positive damage indication ratios for 90%, 95% and 99% confidence levels





Calculated false-positive damage indication ratios for the simulated test cases

	Noise level			Noise level		Noise level			Noise level			
Confidence level Cut depth [mm]	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	<u> </u>
0.5	-	-	-	-	-	-	1.42	0.12	0	0.66	0.04	0
1.0	-	-	-	7.83	1.79	0.08	1.33	0.21	0	0.25	0	0
1.5	-	-	-	7.08	1.67	0.04	0.96	0.08	0	0	0	0
2.0	8.29	1.58	0.04	6.50	1.25	0.04	0.62	0.04	0	0	0	0
2.5	7.91	2.17	0.21	5.66	0.87	0.04	0.12	0	0	0	0	0



51 x 51 measured datapoints



Damage detection results

 $Z_{ij} = \frac{DI_{ij} - \mu_{DI}}{\sigma_{DI}}$

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Experiment (a) 12 800 10 8 600 Y,mm 6 400 4 2 200 0 200 400 600 800 X,mm



FEM $\delta = 0.2\%$

26 x 26 measured datapoints



Damage detection results

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17 x 17 measured datapoints



Damage detection results

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Experiment









13 x 13 measured datapoints



800

600

400

200

200

Y,mm

Damage detection results

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Experiment (a)

400

600

X,mm



3

2

1

0

-1

800





11 x 11 measured datapoints



Concluding remarks

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- Compared to the existing damage detection methods such as MSC and MSCS damage index methods, the advantage of the proposed method is that it requires mode shape information only from the damaged state of the structure
- The obtained results show that the proposed damage index provides reliable information about the location and size of the damage in case of the presence of medium severe damage, relatively accurate measurement data and relatively dense distribution of sensors.
- Last two drawbacks of the method can be overcome by using the latest scanning laser vibrometer systems which allow high-density transverse displacement measurements with a low degree of measurement noise. The major drawback of the method is that the severity of damage has to be relatively high for successful damage detection.
- The obtained results suggest that the proposed method can be applicable not only for laboratory tests but also for practical structural applications.



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Thank You for your attention!







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