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Multiple damage identification in beam structure based on wavelet transform

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Abstract

In this paper a damage identification algorithm for beam structures with multiple damage sites based on wavelet transform method of vibration mode shapes is reported. A complex morlet function of order 1-1 is chosen as a wavelet function. Wavelet transform coefficients serve as a damage indices which are standardized according to statistical hypothesis approach, yielding a standardized damage index distribution over beam coordinate. The peaks with the largest amplitude correspond to the zone of damage. Finite element simulations of proposed methodology involving various artificial noise levels and reduction of mode shape input data points are carried out. Results show that the algorithm is capable of capturing the areas of damage.

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1. Introduction

The need for cheap and simple methods for structural inspection has grown tremendously over past decades. This is due to the fact that complex modern engineering structures, for example, stadiums, dams, skyscrapers, tunnels, etc. have to maintain their integrity and functionality. Failure of these structures leads to tragic consequences as well as heavy material losses. Structural health monitoring (SHM) is an interdisciplinary domain which's prerogative is to evaluate the integrity of structures using non-destructive techniques. SHM can be classified into two categories – the

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ones that detect damage and the ones that not only predict the location and severity of damage but also service life of the structure [1].

While a wide range of vibration-based damage identification mechanisms exist, it is also preferable to employ such damage identification methods that do not require a baseline data of vibration response for a healthy structure, such as natural frequencies, mode shapes and damping. Unfortunately, only few of damage identification methods meet this criterion. One of such methods is Wavelet Transform (WT).

WT is a digital signal processing technique, capable of managing analysis of continuous as well as transient signals [2]. It has gained a wide popularity among many engineering communities and today is used in signal discontinuity detection, image compression and denoising, also in medicine and finance [3]. Several variations of WT exist, namely, discrete WT (DWT) and continuous WT (CWT). While DWT is less redundant in terms of decomposition of original signal into discrete levels, important features of a signal can easily be missed. Thus CWT is a preferable method in SHM due to a more detailed decomposition of a signal [4]. One-dimensional CWT is extensively employed in SHM, for example, single [5-10] and multiple [11,12] fault detection in beams and rotating machinery [13,14].

In this paper, two mill-cut damage sites are located in an aluminium beam using Continuous Wavelet Transform technique. The most promising wavelet function turns out to be complex morlet of order 1-1. Numerically simulated vibrational mode shape signals are corrupted with various levels of noise and reduced by several integer values to simulate the performance of damage detection algorithm in real life situations with different sensor densities. Results suggest that the proposed damage identification algorithm is capable of locating damage even at coarsest sensor grids and noise levels of up to 4 % assuming that appropriate wavelet scale is used.

2. Materials and methods

2.1. Wavelet transform

Mode shapes themselves do not reveal the location of damage, therefore special techniques are required. One of such techniques is Wavelet Transform. Wavelets are special functions $\psi(x)$ with small oscillations such, that their mean is zero. Wavelet transform is a mathematical method to transform the original signal into a different domain where additional data analysis becomes possible. Wavelet transform can be employed to analyse signals f not only in time domain but in space domain as well

$$W_{s,a} = \int_{-\infty}^{\infty} f(x) \cdot \frac{1}{\sqrt{|s|}} \cdot \psi^* \left(\frac{x-a}{s} \right) dx = \int_{-\infty}^{\infty} f(x) \cdot \psi_{s,a}^*(x) dx \quad (1)$$

where asterisk denotes complex conjugation and $\psi_{s,a}(x)$ is a set of wavelet family functions, derived from a *mother wavelet function* $\psi(x)$ by *translating* (parameter a) and *dilating* (parameter s) the $\psi(x)$. Parameter s is a real and positive number. If $0 < s < 1$, the function is expanded, if $s > 1$, it is compressed.

2.2. Damage detection algorithm

Equation (1) was used to calculate CWT coefficients, that are extremely sensitive to any discontinuities and singularities, present in the signal $f(x)$, therefore location of damage due to a sudden loss of stiffness can be detected in those mode shapes that yield large amplitude wavelet coefficients. Damage index for each of mode shapes is depicted as follows

$$DI_{i\ CWT}^n = W_{i\ s,a}^n = \int_L w_i^n \cdot \psi_{s,a}^*(x) dx \quad (2)$$

where L is the length of the beam, w^n is transverse displacement of the structure, n is a mode number, i is number of

grid points in x and direction.

However, mode shapes, measured in real life experimental conditions, are inevitably corrupted by measurement noise, which can lead to false peaks in damage index profiles, thus misleading data interpreter. In order to overcome this problem, it is proposed to summarize the results for all modes. The summarized damage index is then defined as the average summation of damage indices for all modes N , normalized with respect to the largest value of each mode

$$DI_i = \frac{1}{N} \cdot \sum_{n=1}^N \frac{DI_i^n}{DI_{i,\max}^n} \quad (3)$$

According to [15,16], the damage indices, determined for each element, are then standardized

$$SDI_i = \frac{DI_i - \mu_{DI}}{\sigma_{DI}} \quad (4)$$

where μ_{DI} and σ_{DI} are mean value and standard deviation of damage indices in equation (3), respectively. After the standardization a concept of statistical hypothesis testing was applied to classify damaged and healthy elements and to localize damage depending on the pre-defined damage threshold value: choose H_0 (element i of the structure is healthy) if $SDI_i < C_r$ or choose H_1 (element i of the structure is damaged) if $SDI_i \geq C_r$, where C_r is a threshold value. To quantify the reliability of wavelets to identify damage location, a new parameter, called damage estimate reliability (DER) was introduced and calculated as follows:

- The whole interval along the coordinate x of the beam was split into 2 parts:
 - part (a) – zone of damage: $450 \text{ mm} < x < 500 \text{ mm}$, $750 \text{ mm} < x < 800 \text{ mm}$,
 - part (b) – the rest of the beam.
- In each of these parts standardized damage indices were summed and divided by the number of data points in this particular interval, giving average amplitude of SDI (\overline{SDI}) in a respective part.
- DER is equal to average SDI in the zone of damage divided by average SDI in all parts combined, expressed in percentage as

$$DER_i = \frac{\overline{SDI}_i(a) \cdot 100\%}{\overline{SDI}_i(a) + \overline{SDI}_i(b)} \quad (6)$$

In order to compare the sensitivity of the damage identification method to noisy experimental data, a uniformly distributed random noise was added to the numerically simulated mode shapes

$$w_n^{\prime i} = w_n^i \cdot (1 + \delta \cdot (2r - 1)) \quad (7)$$

where prime indicates noisy mode shapes, δ is a level of random noise and r are uniformly distributed random values in the range (0, 1). The levels of noise were chosen to be 0.1 %, 0.3 %, 0.5 %, 1 %, 2 % and 4 %.

It is often not possible to equip the structure with a dense grid of sensors. Therefore an additional study was conducted where numerical mode shape data was divided by integer numbers $p = 1, 2, 3, 4, 5$ and 6, leading to mode shape vectors of length 149, 75, 50, 38, 30 and 25.

DER values for respective SDI were calculated for all p values at each of noise level δ forming a 7 x 6 DER matrix where columns correspond to p values and rows – to noise levels.

2.3. Numerical simulations

The validation of proposed damage identification algorithm is performed by numerical modal analysis based on finite element method (FEM). It is conducted by using the commercial software ANSYS. An aluminium beam with two mill-cut damage sites is considered. Geometrical configuration of the beam is shown in figure 1. The first and second mill-cut damage sites with a depth of 2 mm and width of 50 mm are introduced at a distance of 450 mm and 750 mm from one edge of the beam, respectively. FE model of the beam consists of 2D beam elements. Each node has 3 degrees of freedom, namely translations along the X and Y axes and rotation along the Z axis. The beam is constructed by means of 148 equal length elements ($i = 149$ nodes). The elastic material properties are taken as follows: Young's modulus $E = 69$ GPa, Poisson's ratio $\nu = 0.31$, and the mass density $\rho = 2708$ kg/m³. The damage is modelled by reducing the flexural stiffness of the selected elements, which is achieved by decreasing the thickness of elements in the damaged region of the beam. In total, 11 modal frequencies and corresponding mode shapes are calculated.

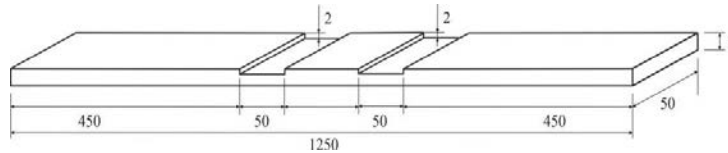


Fig. 1. Geometry and dimensions of tested aluminium beams.

3. Results and discussion

Overall, 78 wavelet functions are tested in terms of DER values for damage identification. These wavelets include all wavelets available in Matlab Wavelet Toolbox menus. As one can see in figure 2, the majority of wavelets (17) yield DER values between 94 % and 95 %, indicating a very good damage localization. Out of these wavelets a complex morlet wavelet of order 1-1 (cmor1-1) is selected for the damage identification as it yields the best DER value of 97.71 % at scale 6.

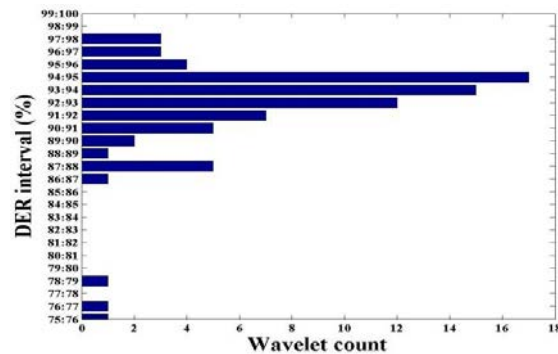


Fig. 2. Maximum DER values for all 78 wavelet functions.

An analysis of cmor1-1 wavelet performance in terms of scale parameter (or scales) is carried out. In total, 64 scales are included in the study. It is notable that damage identification is effective for scales below 10 (refer to figure 3 (a)). This result holds even with a varying sensor grid density, although at $p = 5$ there is a sudden leap in scale (Scales at best DER in figure 3 (a)). Ignoring this leap and setting maximum scale value to 10, a curve Scales* is obtained, since DER vs scale plots are smooth till scale value of about 32. Therefore, only minor deviations of scale at which cmor1-1 wavelet attains maximum DER values are observed.

A DER matrix, composed of DER values for cmor1-1 wavelet for different sensor densities and noise levels is shown in figure 3 (b). It can be seen that out of these two factors the sensor density has the most influence on damage identification results – there is about 10 % drop in DER with p varying from 1 to 6.

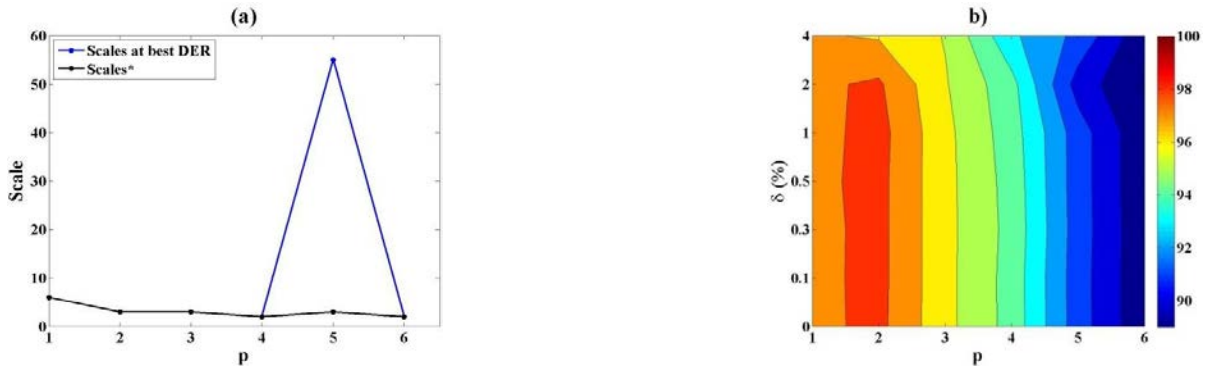


Fig. 3. a) Scale vs p plot; b) DER matrix for DI sum over all modes.

DER vs scale plots at noise levels 0 % and 4 % are shown in figures 4-6 (a) for $p = 1, 3$ and 6, respectively, while SDI distribution along the beam at 0% noise is shown in the same figures (b).

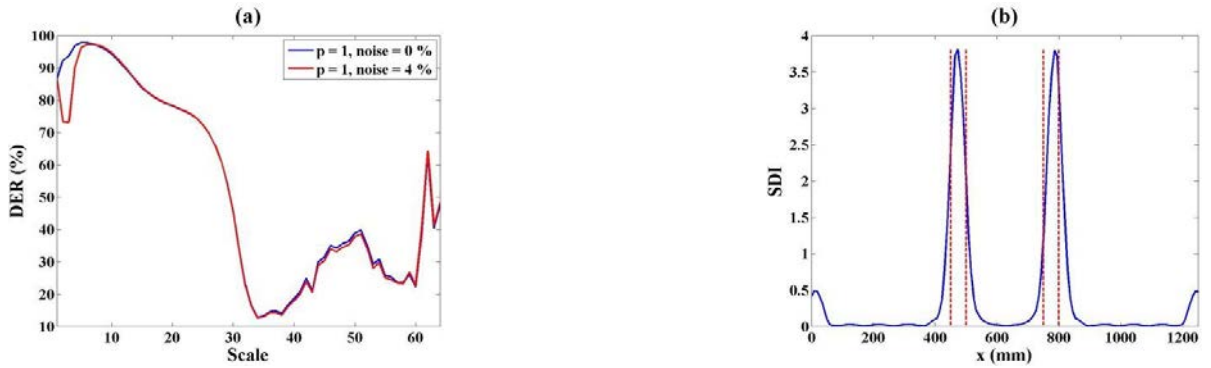


Fig. 4. $p = 1, n = 0 \%$ and 4 %. (a): DER vs scale plot, (b): SDI(x) plot for scale = 6.

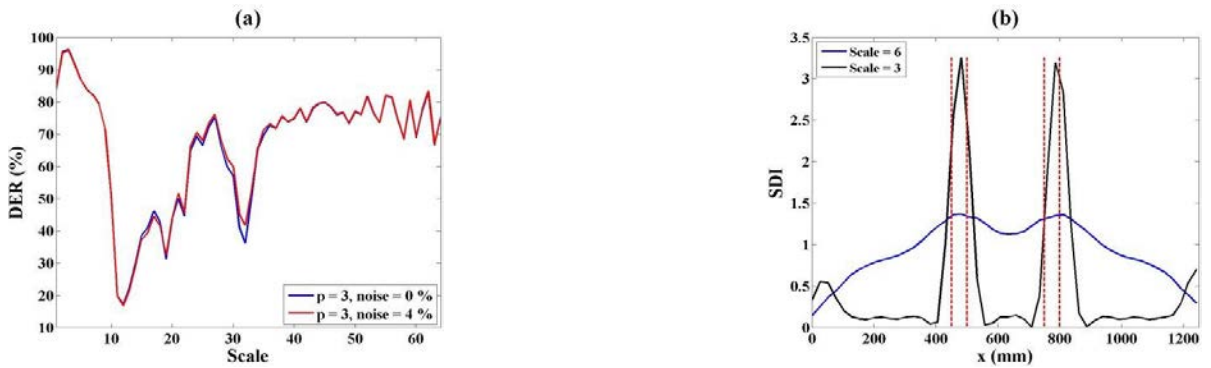


Fig. 5. $p = 3, n = 0 \%, 4 \%$. (a): DER vs scale plot, (b): SDI(x) plot for scale = 6 and 3.

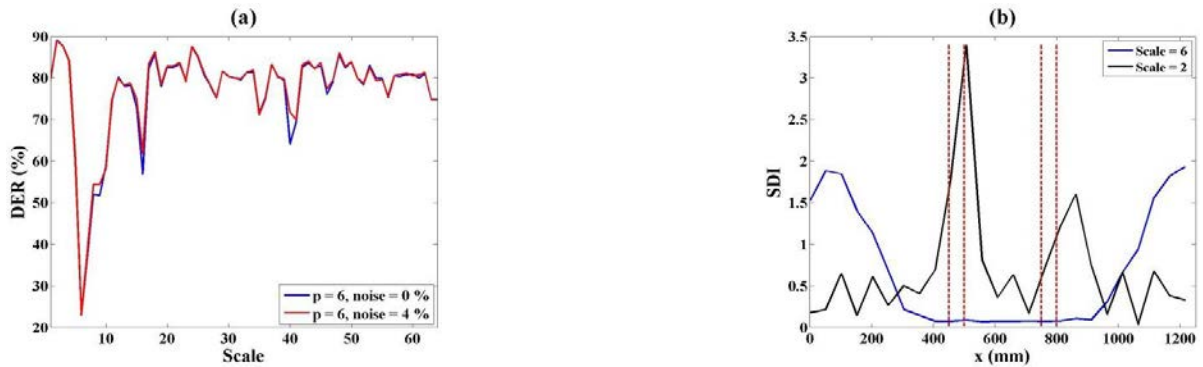


Fig. 6. $p = 6$, $n = 0\%$, 4% . (a): DER vs scale plot, (b): SDI(x) plot for scale = 6 and 2.

It can be seen that for DER plots exhibit some slight differences between no-noise and 4 % noise cases but mainly at some scale intervals. Also, smoothness of DER curves decreases with increasing sensor grid coarsity.

The respective SDI distributions are all shown at scale = 6, set as a reference, since highest DER value is attained at this scale, at original sensor density. At $p = 1$ the highest SDI peaks are convincingly located between two red vertical lines that indicate the zones of damage. At $p = 3$, however SDI are widely spread across the coordinate of the beam, clearly not revealing the damage (at scale = 6). Nevertheless, scale = 3 is the best for this case as indicated by a respective DER vs p plot – one can see that once again two peaks with the largest amplitude are located in the zones of damage. Scale = 6 fails to indicate damage also at the most coarse sensor grid, while scale = 2 is the best for this case. However, both peaks are a bit shifted out of the damage zones.

4. Conclusions

A methodology based on Continuous Wavelet Transform for identification of multiple damage sites in aluminium beam is proposed. Numerical simulations of vibrational mode shapes of the beam polluted with noise of levels up to 4 % and reduced by an integer factor to account for limited sensor density are carried out.

Out of 78 wavelet functions a complex morlet wavelet of order 1-1 is selected due to the highest damage estimate reliability parameter of 97.71 %. In general, the performance of most of the wavelet functions is well above 90 %, suggesting that algorithm works well.

Wavelet scale analysis must be conducted before the identification of damage. In total, 64 scales are tested and the best damage identification results correspond to scales $s < 10$. This trend does not change with increasing coarsity of a sensor grid, however best damage identification results are not attained at a fixed scale for each of sensor grid densities – as sensor grid density changes, so does the associated scale that yields the best results of damage identification.

It is found that algorithm successfully manages to localize both sites of damage irrespectively of noise.

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